# Natural Frequency

The natural frequency is observed when the system is left at any point and the oscillations due to the spring force is observed. (When P(t) is not actively oscillating the system) Thus, the load should be taken as zero, while the system is oscillating with this frequency. Then any frequency that solves the governing equation are the natural frequencies of the system.

Equation 1 is the governing equation of the system. It is assumed to be solvable by separation of variables. Hence, Equation 3 can be written. Equations 1, 2, and 3 can be combined to create Equation 4. For the multi-variable differential case to be valid in Equation 4 each of these derivation will have to be equal to a constant which is taken as .

The solutions ofand for Equation 4 are given in Equations 5 and 6 where These equations will have to be solved using the boundary and initial conditions for each system.

The axial force in the spring can be seen in Equation 7. Using Equations 3, 5, and 6 it can also be simplified to Equation 8.

Writing the equation of motion for the mass on the end of the spring, Equation 9 can be obtained where the acceleration of the cube must be equal to the acceleration of the tip of the spring as given in Equation 10.

Starting to apply the boundary conditions, the first boundary condition is that the root of the spring will always be stationary since it is connected to a wall, i.e. . Using this boundary condition Equation 11 can be followed to find .

Applying the initial condition that the acceleration must be 0 when the time is 0 (); Equation 12 can be followed to obtain the three roots given in Equation 13.

Following the non-trivial solution , we can obtain Equation 14 and in return the formula for the natural frequency given in Equation 15.